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A.5: Special Relativity

• https://openstax.org/books/university-physics-volume-3/pages/5- ~~introduction~~ • https://youtu.be/bJMYoj4hHqU?si=H\_nz648IrI92zN0C ~~(Frames of reference)~~

• https://www.youtube.com/watch?v=JqwxQvq8IH0&list=PL2RRoMlng ~~3gpCG4lWX1\_BG-9D83KsKFBj&index=7 (Lorentz Transformation)~~ • https://www.youtube.com/watch?v=aQQXWqkGvB0&list=PL2RRoMln ~~g3gpCG4lWX1\_BG-9D83KsKFBj&index=6 (Time Dilation)~~ • https://www.youtube.com/watch?v=9nr9pfyD71w&list=PL2RRoMlng 3gpCG4lWX1\_BG-9D83KsKFBj&index=1 (A.5 with Worksheets)

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Relativity *Special and General*

• **Relativity** is about relative motions and physics as experienced when observer and a physical phenomena do not follow the same measurement coordinates.

• **SPECIAL theory of Relativity (STR)**: About what happens to physics when relative speed of two sets of coordinates move closer to speed of light.

• **STR** (as well as GTR) are a paradigm shift. (NOS)

• It is **NOT the same** as the General theory of relativity (GTR) [Misconceptions]

• We shall NOT study the GTR.

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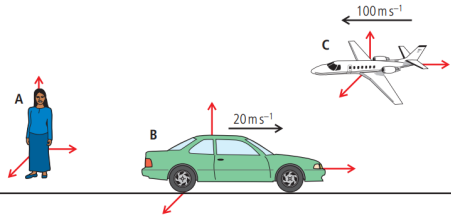
Special Relativity

• All the mechanics you studied till now was “experiential”. • Einstein’s Relativistic mechanics, however, is not.

• Involves very high speeds (comparable to speed of light ��) unachievable for practical objects even today

• Speeds the fraction of speed of light �� = 3×10!����"# in air/vacuum • Special Relativity is a “paradigm shift” in understanding motion

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Terminology

• Event: Some change in

time and space

• Observer: Someone who

measures physical quantity

related to an event.

• Frame of reference: System (coordinates) of measurement

• An observer can only make measurement in his/her own system

• E.g.

• **Event**: observer moving/time passing • **Observers**: Persons at (A), in car (B), in plane (C) • Frame of reference: Point A, Car (B), plane (C)

• An observer can only make measurement in his /her own system

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Measurements and

Reference Frames

• Convention: Velocities add when

coming together.

Left ⇒ negative velocity (�� < 0)

Right ⇒ positive velocity (�� > 0)

• As measured by **A:** • Velocity of A: 0

• Velocity of car: 20 ����"#

• Velocity of plane: − 100 ����"#

• As Measured by **B**: • Velocity of A: −20 ����"# • Velocity of car: 0

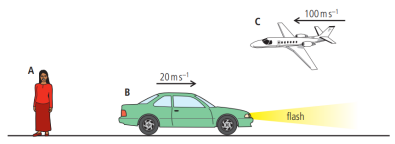
• Velocity of plane:

− 120 ����"#

• As measured by **C**:

• Velocity of A: 100 ����"# • Velocity of car: 120 ����"# • Velocity of plane:0

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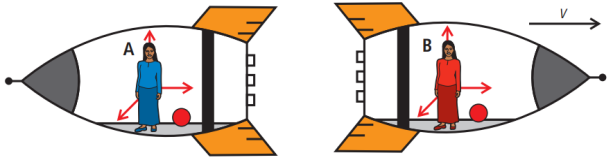
• Learning outcome:

• For measurements sake, a “frame of reference” is just a set of coordinates

• Same quantity measured from different ‘frames of reference’ will give you different measurements when they are moving with relation to each other. • Thus, relative movement poses complications.

• *Earth is a frame of reference, and moon is another*

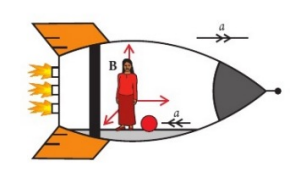
*Something that is stationary on earth will not be with respect to another.* 11/10/25 Dr. Vaibhav Kaware 8

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• E.g. Earth’s gravity, rotation pose complications to references • Hence, our systems will be in free space (devoid of gravity, rotation etc.) • Case 1: Rocket B moves towards right with some ‘**constant**’ **speed** �� • **no acceleration** ⇒ **Inertial** frame of reference • *Ball on the ground won’t move unless kicked*, in ***either of the rockets***.

• Newton’s laws are same individually in both frames.

Ball on ground will not move without an external unbalanced force acting on it. 11/10/25 Dr. Vaibhav Kaware 9

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• E.g. Earth’s gravity, rotation pose complications to references • Hence, our systems will be in free space (devoid of gravity, rotation etc.) • Case 2: Rocket B is accelerated

• Ball on ground will move without kicking (“seems” Newton’s laws are broken) • … as a reaction to force from floor. (Newton’s laws are still followed) • ***Accelerated frame is called “NON-inertial” frame of reference.*** • *We study only inertial frames (frames moving with constant speed* ��*)*

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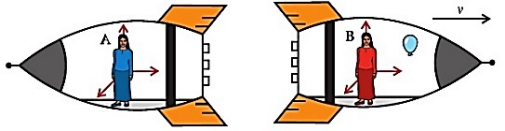
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Galilean Relativity (for *inertial frames*)

• Principle of relativity

“*Basic Laws of Physics are the same in all inertial (uniformly moving) frames of reference (individually)”*

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Galilean Relativity (for *inertial frames* only)

• Measurement: Position of blue balloon

• Coordinates measured by A ∶ x

• Coordinates measured by B ∶ x′

• ��: constant speed of separation

• NOTE: ***time*** �� ***is same in both frames***

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Length in Galilean

Relativity

• Length is difference between positions

• �� = ��$ − ��# = ��$% + ���� − ��#% + ���� = ��$% − ��#′

• Length is invariant in inertial frames (Galilean relativity)

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Velocity in Galilean

Relativity

• Length is difference between positions

• �� = ��$ − ��# = ��$% + ���� − ��#% + ���� = ��$% − ��#′

• Length is invariant in inertial frames (Galilean relativity)

• Velocity measured from A, �� = ��% + ��

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Acceleration in

Galilean Relativity

• Length is difference between positions

• �� = ��$ − ��# = ��$% + ���� − ��#% + ���� = ��$% − ��#′

• Length is invariant in inertial frames (Galilean relativity)

• Velocity measured from A, �� = ��% + ��

• Bird accelerates from ��#% to ��$′ in time Δ��

'( as measured in A: �� =&#" )\* " &!" )\*

• Acceleration in B ��′ =&!" "&#"

'(= ��′

• Hence, ***acceleration is also invariant in Galilean relativity (inertial frames)*** • ⇒ Newton’s second law applies as is in inertial frame

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*Enter ….. Special Theory of Relativity (STR)* 11/10/25 Dr. Vaibhav Kaware 18

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Convention and terminology

• *Measurements* made *in the same* (usually the moving) *frame of reference* where the event takes place, are ‘**proper**’ measurements • We shall append symbol ′ (prime) to denote these quantities

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Need for the STR (Problems in the heaven!)

• Galilean relativity is logically sound and practical.

Applies to objects we see around.

• Things like electromagnetic waves couldn’t fit in these descriptions (thanks to Maxwell)

• Maxwell explained how light travelled through vacuum as wave of electric and magnetic oscillations (Electromagnetic wave)

• Theory by maxwell predicted the same speed �� =����������≈ 3×10! ����"# • It involves only constants of medium.

• Has not scope for inclusion of relative motion predictions at *c.* • Speed of light measured till then (circa ~1900 AD) �� ≈ 3×10! ����"# PPT - Chapter 27: Light PowerPoint Presentation, free download - ID:1886400 (slideserve.com) Speed of light [3 of 4] measured by Romer | PPT (slideshare.net)

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Speed of light [3 of 4] measured by Romer | PPT

(slideshare.net)

michealsonss-method-velocity-light *~1675 AD*

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Michaelson's experiment to measure the speed of light ��

https://www.sciencetopia.net/physics/

michealsonss-method-velocity-light

Speed of Light Experiment by Michelson

(youtube.com)

***~1926***

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Speed of Light

Experiment by

Michelson

(youtube.com)

https://www.youtube.c

om/watch?v=B\_T\_Xi\_bd

1c&t=13s

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Speed of light in

Galilean Relativity

• In Galilean relativity �� = ��% + ��

• Which for speed of light in above case, means

• �� = ��% + �� ; �� = 3×10! + 2×10! = 5×10! �� ��"#

• However, this clashes with Maxwell’s speed of light equations, and … • Experiments had proven Maxwell correct.

• �� ≈ 3×10!����"# whether source is moving or not.

• Maxwell’s momentum of light �� = ��/�� does not fit classical definition (�� = ����) • Einstein showed how to fit all these together in special relativity

PPT - Chapter 27: Light PowerPoint Presentation, free download - ID:1886400 (slideserve.com)

Speed of light [3 of 4] measured by Romer | PPT (slideshare.net)

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STR postulates

• “�������� ���� ��ℎ���������� ������ �������� ���� ������ ���������������� ������������ ���� ������������������”

• “���������� ���� ������ℎ�� ���� ��ℎ�� �������� ���� ���������������� ���� ������

���������������� ���������� ������������������”

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***Galilean Case***

• Ball bounces to travel different distances in two cases (a) & (b) • Case (a) Time of travel between bounces ����=����

����

• Case (b) for the bouncing ball, total velocity will be greater: vector sum ��%&' = ��( + ��)%\*++&,

• Case (b) When moving, ���� =����������������

��������; distance measured by stationary

observer is greater but so is the velocity observed by stationary observer. • Hence, time remains same (����= ���� ) when distance increases 11/10/25 Dr. Vaibhav Kaware 26

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***Special Relativity Case***

• **Light** travels different distances in these two reference frames (a) and (b) • (a) Time of travel between bounces ����=������(��. is the Proper time ��′) • (b) From moving reference frame, ���� =����������������

��;

• For light, there is not vector sum so �� + �� = �� (2nd postulate)

• Hence, to cover more distance with same speed �� you need ***more time*** ⇒ ����> ������ > ��/ • Time in (relatively) moving FOR (a), when measured by an observer in RF (b) passes slower, compared to time as measured by observer in FOR (a) (proper time) • **Improper time interval is greater than the proper time interval** 11/10/25 Dr. Vaibhav Kaware 27

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**Galilean relativity**

• Time in an inertial FOR measured by an external fixed observer remains same as when measured by the moving observer, since measured velocity of object can change.

• �� = ��. ��%= ��

**Special relativity**

• Time in an inertial (moving) FOR (a), when measured by an observer in (stationary) FOR (b), passes slower compared to time measured in RF (a)

• ��/ < ��. (proper time ��% < �� )

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The problem of synchronization

• ������������ ��ℎ���� ������ ��������ℎ�������������� ���� ������ ���������� ���� ������������������ ������ ������ ��������ℎ�������������� ���� ��ℎ�� ����ℎ����.

• Time is synchronized since light from your hand reaches both observers at the same time. Their clocks start at the same time.

• ***For the moving observer,*** left distance is

more, since left is (relatively) moving in the

direction of light itself

• And right distance is less since light is

moving towards right

• Hence, their clocks start at different times

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• Think of two sprinklers separated by some distance & **switched on at the same time (simultaneously)**.

Simultaneity: An Analogy

• If a person stands **exactly in the middle**, and another one travels **on a motorcycle** (at reasonable speed) between them,

• Which sprinkler’s water will get the stationary person wet first? • Which sprinkler’s water will get the person on bike wet first?

• Problem with STR’s simultaneity is that we fail to understand that **no information** reaches observer, “before he knows there is information”.

• Unlike in sprinkler case, we know water’s position/velocity etc. even before the water gets them wet.

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Simultaneity: From different FOR

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Transformations in Einstein’s relativity

• Since synchronization is not possible in moving frames, the only thing we can do, is find a way to transform measurements from one frame to other as measured by them individually.

• Galilean transforms don’t work for (speed of) light

• Transforms in Einstein’s relativity are called the “Lorentzian transforms” • Lorentzian transform is multiplicative factor ��:

�� =��

�� − ����

����

• Dimensionless,

28.2: Simultaneity and Time Dilation - Physics LibreTexts https://byjus.com/physics/derivation-of-lorentz-transformation/

• �� is velocity of frame �� with respect to (wrt) ��, ��: speed of light 11/10/25 Dr. Vaibhav Kaware 33

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**Galilean Relativity**

• ∵ �� = ��% + ����

• ∴ Moving frame position ��%= �� − ����

• Length Δ��′ = Δ��

• Time instant ��%= �� • Time interval Δ��%= Δ��

**Einstein’s Special Relativity**

• ��%= �� �� − ����

• Δ��%= ��(Δ�� − �� Δ��)

• ��′ = �� �� − \*1

2!

• Δ��%= �� Δ�� − \* '1

2!

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https://upload.wikimedia.org/wikipedia/commons/9/98/Loren

tz\_boost\_x\_direction\_standard\_configuration.svg

**To revert the transform equations,**

**Swap primed and unprimed quantities and reverse the sign**

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Data Booklet

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Data Booklet

**Formulae with a** ���� **and/or** �� **are Relativistic, the others**

**are Galilean**

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#"%!&~~!~~=#

#" !×#() !

\*×#() ~~!~~

=#

#"+,= 1.34 …… (��)

• ��%= �� �� − ���� = 1.34 100 − 2×10!×4×10"! = 123.43 �� ……. (��) 2!= 1.34× 4×10"! − $×#7)×#77

• ��%= �� �� − \*1

2!

• = 1.34× 2× 10"! − #7)×#7!

8×#7#-= 2.68× 10"! − #8×10"9

• = 2.68× 0.01 − 0.11 ×10"9= −5.69×10": ��…. (��) 11/10/25 --Dr. Vaibhav Kaware 39

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Simultaneity

• Two events which occur simultaneously in one frame, are not so in another moving reference frame.

• Lightening strikes simultaneously at

4 ���� on two trees 5 km apart

• For someone in rocket moving at 0.9 ��

from tree 1 to 2 it won’t be!

• Calculate time between the two

events as observed from the spaceship.

• ��/= �� �� − 01

2) ; �� = 2.3 ; first lightning strike as observed from spaceship occurs at position �� = 0 ∴ ��3/= 2.3× 4 ���� − 4.6 2×4

2)= 9.2 ����

• ��9/= 2.3× 4 ���� − 4.6 ��× :×34\*

2��= −25.4 ����

• To observer in the rocket, seconds tree was hit 9.2 − −25.4 = 34.5 ���� earlier. • Rocket travelling towards tree 2, hence light travels less time to the rocket ⇒ earlier

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• Lightening strikes simultaneously at 4 ���� on two trees 100 �� apart • For someone in rocket moving at 0.8 �� from tree 1 to 2 it won’t be! • Calculate time between the two events as observed from the spaceship. • ��/= �� �� − 01

2) ; �� = 1.67

• first lightning strike as observed from spaceship occurs at position �� = 0 • ∴ ��3/= 1.67× 4 ���� − 4.; 2×4

2)= 6.68 ����

• ��9/= 1.67× 4 ���� − 4.; ��× 34)

2��= −15.5 ����

• To observer in the rocket, seconds tree was hit 6.68 + 15.5 = 2.218 ���� earlier. • Rocket travelling towards tree 2, hence light takes less time to travels to the rocket ⇒ event occurs earlier

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Time Dilation

• Light is switched on in FOR ��′ initially at time ��#′, and then again at time ��$% (��’ is the proper FOR)

• ∴ Δ��′= ��$′ − ��#′

• Other observer moving at �� measures times ��# & ��$. ∴ Δ�� = ��$ − ��# • ��# = �� ��#% + ����

���� & ��$ = �� ��$% + ����

���� ∴ Δ�� = ��(��$′ − ��#′)

• Thus, intervals of time scale as ���� = ������′ **(**OR �� = ������**)** • Since �� > 1 always,

time measured from external

FOR will always

be ***dilated (longer)*** than proper time interval.

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• Δ�� = 1 �� ∴ Δ�� = ��Δ��′

• �� = 1.7 ; ∴ Δ�� = 1.7 ��

• �� =#

#"%!&~~!~~=#

#" 7.:!=#7.?#= 1.4

• Δ�� = ��×��%

• = 1.4×2 = 2.8 ��

• �� =#

#"7.88!= 7.089

• ��# !

= 7.089×30

• = 212.7 ��

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• (a) Proper time interval Δ�� is the time measured

by an observer at rest relative to the event being

observed.

• Hence, observer in the rocket measures the

proper time ��/= 2 ��

• (b) �� =3

3<4.;)= 1.667

• �� = ��×��/= 1.667×2 �� = 3.33 ��

• Simultaneous events occur at same time separated in space • Time dilation occurs at same position separated by time (time interval) 11/10/25 --Dr. Vaibhav Kaware 47

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Length contraction

• Length of the rod in moving FOR

• Δ��%= ��$% − ��#% Proper length, since rod is not moving wrt observer in ��′

• For another observer (in stationary/fixed frame ��)

• ��#%= ��(��# − ����) & ��$%= ��(��$ − ����)

• i.e., Δ��%= ��$% − ��#%= �� ��$ − ��# = �� Δ��

• ���� =��������% ���� �� =��������

• Length measured from external frame appears contracted 11/10/25 --Dr. Vaibhav Kaware 48

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Length contraction

• Length of the rod in moving FOR

• Δ��%= ��$% − ��#% Proper length, since rod is not moving wrt observer in ��′

• For another observer (in stationary/fixed frame ��)

• ��#%= ��(��# − ����) & ��$%= ��(��$ − ����)

• i.e., Δ��%= ��$% − ��#%= �� ��$ − ��# = �� Δ��

• ���� =��������% ���� �� =��������

• Length measured from external frame appears contracted 11/10/25 --Dr. Vaibhav Kaware 49

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• Since rod is in the space station and the measurement is made in the space station itself, that is the proper length ��%= 44 ����,

• �� =@"A=@(A

• �� =#

#" (.)\*& !

&~~!~~

• Hence, �� =CC

=#

#"7.!B!= 1.793

#.:8B= 24.54 ����

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• �� =#

#"%!&~~!~~=#

#" 7.:!=#7.?#= 1.4

• Spaceship �� is the proper system since rod and the

observed are in spaceship B

• Proper length ��7 = 2 ��

• ∴ length as measured by observer in spaceship A is

• �� =@(A=$#.C= 1.43 ��

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• �� =3

3<,)-~~)~~=3

3< 4.;)= 1.67

• Earth is the proper system

• Hence **(a)** Δt=>?@A =BCD@>EF=

DG==B=: HI

4.; 2

• = ��. ���� ��

• **(c)** Improper time ���� = ��/��

• Δt =BDE

7.! F= ��. ���� ��

• *Cannot apply time dilation since events (of measuring the distance) do not occur at the same point n space.*

• **(b)** Here, ��4 = ��&.%)J = 5 ���� • ∴ Improper distance as measured from the rocket, �� = ��4/��

• ∴ �� =:

3.KL= 2.99 ���� ≈ 3 ����

• contracted

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Velocity Lorentzian transform

• ��%=��"��

��" ����

����

• Here, �� is the velocity measured

from the stationary FOR, ��

• … and ��% the same velocity as

measured from the moving FOR ��′.

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Velocity Lorentzian transform

• ��%=��"��

��" ����

����

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• Stationary observer: Astronaut (unprimed external observer)

• Assume one ship �� (0.8 ��) as moving FOR (primed-proper FOR), and then find speed ��′ of the other ship �� in FOR of first ship(moving FOR). Hence,

• �� = 0.8 �� & �� = −0.9 �� (B is moving to the left)

• ��/=��<��

��< ����

��~~��~~

• ��/= <4.6 <4.; 2

3< 12.4 -×2.7 -

-~~)~~

• ��/= − 3.L 2

4.99

• ��/= −0.988 ��

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Speed of light

• ��%=��"��

��" ����

����

• For �� = ��,

• ��%=��"�� ��" ����

����

=��"�� ����3����

����

=2"\*

�� &3%

����

= ��

• Thus, �� is invariant under the Lorentzian transform (does not depend on �� of the observer)

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• Stationary FOR �� is earth

• One of the particles is moving FOR, hence, �� = 0.9 ��

• The other is moving in opposite direction to �� with same speed, hence, • �� = −0.9 ��

• ��%=��"��

��" ����

����

• ∴ ��%= "7.82"7.82 #"(3)(.,& ×(.,&

&~~!~~

= − #.! 2

#)7.!#= −0.9945 ��

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• Earth: stationary FOR �� (unprimed)

• Spaceship: Moving FOR ��′; Hence, �� = 0.6 �� (�� assumed to right) • Meteorite ‘approaches’ spaceship hence, �� = −0.5 �� • To find velocity of meteorite ��′ as measured from spaceship

• ��%=&"\* #" 7%

&~~!~~

= "7.?2"7.92 #"~~3(.8&×(.-&~~ &~~!~~

= − #.#2

#" "7.B= − #.#

#.B�� = −0.8462 ��

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• Moving frame ��′ is the spaceship.

• Velocity of the probe is with respect to (wrt) moving spaceship S%, hence “proper” velocity of probe is ��′ = 0.5 ��

• (In previous examples, both velocities were wrt ground ��) • To find �� of the probe (as measured from stationary FOR - ground)

• Since ��′ =&"\* #" 7%

&~~!~~

, reverting the velocity transform for ��, we get

• �� =&")\* #)7~~"~~%

&~~!~~

=7.?)7.! 2 #)~~(.8&×(.)&~~ &~~!~~

=#.B2

#.C= 0.93 ��

https://www.youtube.com/watch?v=JqwxQvq8IH0&list=PL2RRoMlng3gpCG4lWX1\_BG-9D83KsKFBj&index=7 from CBS 11/10/25 Dr. Vaibhav Kaware 61

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• Earth: stationary FOR ��

• Car: Moving FOR ��′

• Hence, �� = 0.8 ��

• Fly flies ‘in the same direction as’ spaceship hence, �� = 0.7 �� • To find: ��′

• ��%=&"\* #" 7%

&~~!~~

=7.:2"7.!2 #"~~(.)&×(.9&~~ &~~!~~

= − 7.#2

#"7.?9= −0.2273 ��

• Fly will be left behind the car at a relative speed of −0.23 ��. 11/10/25 Dr. Vaibhav Kaware 62

**~~D~~r. Vaibhav K~~a~~ware** 

What really travels near the speed of light?

• �� particles (muons) are produced in upper atmosphere (~10 ���� above) when the (pions) �� particles from the cosmic rays, decay. (π → ��)

• Speed of ��, �� = 0.98 ��

https://www.youtube.com/watch?v=nxQ0kz\_qWmc&t=198s

• Half-life 1.6 ���� ⇒ Half of initial number will decay in 1.6 ����. 

• Stationary muon’s mean lifetimes 2.2 ����

• Without STR, time required for muons to travel is �� =!"" $

".&'×(\*×!"#)= 34 ����

• Number of muons reaching earth’s surface from ℎ = 10 ���� should be negligible, since time available for them to travel is much less than time required to travel [2.2 ���� & 1.6 ���� ≪ 34 ����]

• With STR, �� = 5 for �� = 0.98 ��

• From earth’s FOR, time of travel to surface is dilated to �� = ����" = 5×1.6 = 8 ���� • From muon’s FOR, length scales as �� =,$-=!-×10. = 2 ����, and they will take /×!"%

".&'× \*×!"#= 68 ���� much greater than time available T!// = 1.6 ���� • Hence, much more than half the initial number reach the ground

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The space-time fabric

• The space-time interval is invariant in

Einstein’s relativity.

• Δ��: Total time of light’s travel

• By Pythagoras’ theorem,

• ��$ =2 '( $

$− '1$$

• For a different speed ��,

• ��$ =2 '(" $

$− '1" $

$

• ∴2 '( ��

$− '1��$=2 '(" ��

$− '1" ��

$

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**Dr. Vaibhav Kaware** 

The space-time fabric

• The space-time interval is invariant in

Einstein’s relativity. ∴

• ��Δ�� $ − Δ�� $ = �� Δ��′ $ − Δ��′ $

• Quantity ���� �� − ���� is the space-time

***interval (STI)***

• For stationary light clock, Δ�� = 0

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• ***Reconsider***, two FOR �� & ��′ coincide at �� = 0 and then separate at 0.9 �� relative speed. Observer in �� sees a

balloon pop �� = 5 �� at time �� = 10"!��. Calculate the space-time interval (������) for each of the observer

towards the event

• For �� = 0.9 ��, �� = 2.3

• ��%= �� �� − ���� = 2.3(5 − 0.9×3×10!×10"!

• ��%= 5.28 �� & proper time ��′ (in frame ��)

• ��%= �� �� − \*1

2!= 2.3 10"! − 7.8 �� × ?

• ��′ = −1.15×10"! ��

• ∴ ������ in �� ��HIJ = ���� $ − ��$

2��

= 3×10!× 10"! $ − 5$ = −16 ��$ &

%= ����% $ − ��% $

• ��HIJ

= 3×10!× −1.15×10"! $ − 5.28$ = −16 ��$

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**Dr. Vaibhav Kawar~~e~~** 

• �� =!

!1&'(~~'~~=!

!1 '×\*$# '

%×\*$# ~~'~~

∴ �� = ��. ������ …… (��)

• ��2= �� �� − ���� = 1.34 [

=!

!1"+

100 −

• Calculate STIs for the above problem

2×10'×4×101' ]

• ��HIJ = 3×10! × 4×10"! $ − 100$ • ��HIJ = −9856 ��$

• ��HIJ = 3×10! × −5.69×10": $ − 123.43$ • ��HIJ = −9856 ��$

• ∴ ��2= ������. ���� �� ……. (��) • ��2= �� �� − 34

5'= 1.34×

2×101' − /×!"#×!""

5'

• = 1.34× 2× 101' − /×!"#×!"' &×!"\*,

• = 2.68× 101' − /&×1016 ��2= −��. ����×����1�� ��…. (��)

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Space-Time Diagrams 

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(STD)

18

• **NOS:** Theory is easier to understand 16

when expressed in diagrams

14

• **Space-time diagrams** are an alternative way to solve STR problems.

12

y=0.1 x y=0.2 x y=0.5 x y=1 x y=2 x y=3 x y=4 x y=5 x y=6 x

• **Space-time connection** is difficult to

y

10

comprehend

8

• **Space-time diagrams** make is easier to

6

visualize

4

2

0

0 2 4 6 8 10 12 14 16 18 20

x

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Space-Time Diagrams 

**Dr. Vaibhav Kaware** 20

(STD)

18

• **NOS:** Theory is easier to understand 16

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• **Space-time diagrams** make is easier to

8

visualize

6

• These diagrams require understanding of

4

graphs

2

• 1. Higher the gradient steeper the line

0

0 2 4 6 8 10 12 14 16 18 20 x

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Space-Time Diagrams (STD)

• **NOS:** Theory is easier to understand

when expressed in diagrams

• **Space-time diagrams** are an alternative

way to solve STR problems.

• **Space-time connection** is difficult to comprehend

20

18

• **Space-time diagrams** make is easier to

16

visualize

14

12

• These diagrams require understanding of y

10

graphs

8

• 1. Higher the gradient steeper the line

6

4

• 2. Exchange of axis reflects graph around 45° line

2

0

• 3. Gradient get reciprocated

y=0.2 x

y=0.5 x

y=1 x

y=2 x

0 2 4 6 8 10 12 14 16 18 20 x

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Space-Time Diagrams (STD)

• Gradient of graph reciprocates when axes are

exchanged

• �� ='K

'( velocity

• ��%='(

'11/velocity & velocity=#L%

• Gradient �� has dimensions of velocity (����"#)

• Gradient ��′ has dimensions (1/����"#)

• Larger ��% ⇒ lower velocity &

Smaller ��% ⇒ Higher velocity

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Space-Time Diagrams (STD)

• Gradient ��′ becomes dimensionless if we plot

���� instead of just �� on the �� axis

• gradients = 1 & 2 respectively

• Both graphs have gradients with no dimensions

• ��-axis *is time* in terms of ���������� (×��������)

• 2nd graph corresponds to **lower** velocity of

object (higher gradient)

(since time is on y-axis and displacement on ��)

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Space-Time Diagrams (STD)

• In Relativity’s Space-Time Diagram, �� axis is ����

instead of ����

• STD compares displacement of particle �� (= ����)

with displacement of photon (����)

• Photon has gradient �� = 1 on this graph

• The space-time line is called ������������������

• All speeds of objects have worldlines ‘above’ the

gradient �� = 1 (photon’s) worldline (�� < �� always)

• In time ��3 object travels distance ����3 (��-axis) and in

same time, photon would have travelled distance

����3

• Gradient of worldline is ������ �� =0)8

2)8=02

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Space Time Diagrams (STD)

• Every point in STD is an event

• Vertical line ⇒ position fixed in

space (all 3 dimensions)

• Horizontal line (fixed in time:

imaginative, unreal)

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Space Time Diagrams (STD)

• Every point in STD is an event

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• **Single** straight line ⇒ motion

(worldline) of one object

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Space Time Diagrams (STD)

• Every point in STD is an event

• Vertical line ⇒ position fixed in space

(all 3 dimensions)

• Horizontal line (fixed in time:

imaginative, unreal)

• **Single** straight line ⇒ motion

(worldline) of one object

• **Two** straight lines ⇒ motion of two

(worldline) objects

• **Single** set of axes ⇒ motion observed

within the same FOR

• **Two** sets of axes ⇒ motion observed

from different FOR 11/10/25 --Dr. Vaibhav Kaware 79

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Summary: Space Time Diagram

• Visual representation of events in space at different times.

• It can show the geometry behind phenomena like time dilation and length contraction without using mathematical equations.

• Time (×��) is represented on the vertical axis and space (3D) on the horizontal axis. • The bottom of the diagram (3rd & 4th quadrants) represents the past, or early times & the top (1st and 2nd quadrants) represents the future, or later times. • Each point on the graph, is called an event and an event represents a unique point in space and time.

• An object's world line represents the history of an object's location over time as a line or curve.

• The slope of the world line indicates how fast the object is moving • **Faster** objects having **Less sloped** lines.

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SPACE-TIME DIAGRAM (STD)

• STD has (at least) two sets of axes

• One belongs to fixed frame �� with

axes ���� and ��, and

• The other belongs to moving frame

��% with axes ����′ and ��%

• We drop perpendiculars to get

coordinates on the S axes, while

• We draw parallels to other axis to

get required axis coordinates.

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https://physics.nyu.edu/~ts2/Animation/special\_relativity.html#

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SPACE-TIME DIAGRAM (STD)

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• Events that occur at same place

as seen in frame ��′

• Same event is not at the same

place as seen from the other

FOR ��

• A person sitting in one place is

not moving for people in train,

but

• … is getting displaced as viewed

by someone on the platform

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• Events that occur at same place

as seen in frame ��′

• That event is not at the same

place as seen from the other

FOR ��

• Events 3 and 4 occur at same

time is FOR ��′ but not as seen

from the other FOR ��

• E.g. Loss of simultaneity is STR

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Simple worldlines

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• tan �� =\*2=B?= 0.6

• �� = 0.6 ��

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**~~D~~r. Vaibhav Kaware** 

• tan �� =\*2=B?= 0.6

• �� = 0.6 ��

• Since �� =�� ��" ������

• �� =#

#"((/N O)!

, and tan �� =����,

http://www.trell.org/div/minkowski.html

https://www1.phys.vt.edu/~takeuchi/relativity/notes/section15.html

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STD for the invariant space time interval

• The space-time interval is invariant in

Einstein’s relativity.

• ��Δ�� $ − Δ�� $ = �� Δ��′ $ − Δ��′ $

• Quantity ���� �� − (����)�� is the space

time ***interval***

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